Section 9
Legislative bargaining

Juan Dodyk

Gov 2006 – Harvard

March 26, 2020

Plan for today

– Go over the details of Austen-Smith and Banks' model of coalition formation.

– Go over the details of Dziuda and Loeper's model of dynamic bargaining with an endogenous status quo.

– If there is time, discuss the implications of legislative bargaining and organization for collective choice.

Austen-Smith and Banks (1988)

Three parties, L, M, R, with ideal points $p_L < p_M < p_R$, and seat shares w_L, w_M, w_R. We assume $\max\{w_L, w_M, w_R\} < \frac{1}{2}$. (Otherwise there is nothing to analyze.)

To form a government you need a majority (i.e., a coalition of at least two parties). The government chooses a policy $y \in \mathbb{R}$ and distributes rents G among the parties in the coalition. Preferences of party i are represented by $u_i = -(y - p_i)^2 + g_i$.

Bargaining protocol:

1. The party with the largest seat share proposes a coalition, a policy y and an allocation of G. If the members of the coalition accept, a government is formed.
2. Otherwise, the second largest party makes a proposal.
3. If that fails, the third largest party makes a proposal.
4. If that fails, a "caretaker" government forms and gives each party utility 0.

Result

For any ordering of w_L, w_M, w_R, what happens is that the largest and the smallest parties form a coalition. (In fact, the thing that matters is the order in the bargaining protocol, not the seat share per se.)

How do we prove this? Austen-Smith and Banks do it by brute force: they consider every ordering.

- There is some symmetry: L and R are exchangeable, but the place of M matters.
- There are thus 3 cases: $w_M > w_L > w_R$, $w_L > w_M > w_R$ (which we did in class) and $w_L > w_R > w_M$.

Let's do $w_L > w_R > w_M$ here.

Stage 3: M is the formateur

Party M needs to choose a party j to form a coalition, either L or M, and choose y and $g_M, g_j \geq 0$ such that $g_M + g_j \leq G$.

The problem is to

$$\begin{aligned} \max \quad & -(y-p_M)^2 + g_M \\ \text{s.t.} \quad & -(y-p_j)^2 + g_j \geq 0 \\ & g_M + g_j \leq G \\ & 0 \leq g_M, g_j. \end{aligned}$$

Clearly $g_M + g_j \leq G$ will bind, because increasing g_M is good. Hence $g_M = G - g_j$. We have $g_j \geq (y-p_j)^2 \geq 0$ and $g_j \leq G$. The former will bind, because the lower g_j, the higher g_M. Hence we have $g_j = (y-p_j)^2$, and $(y-p_j)^2 \leq G$. The problem becomes

$$\begin{aligned} \max \quad & -(y-p_M)^2 + G - g_j = -(y-p_M)^2 - (y-p_j)^2 + G \\ \text{s.t.} \quad & -(y-p_j)^2 \geq -G. \end{aligned}$$

If we ignore the constraint, we get the solution $y = \frac{1}{2}(p_M + p_j)$.

Does this satisfy the constraint? We are assuming that the "caretaker" can choose a policy $\bar{y}$ and transfers g such that $-(\bar{y} - p_i)^2 + g_i = 0$, which, summing, implies $-\sum_i(\bar{y} - p_i)^2 = -\sum_i g_i \geq -G$. Now, $-(y - p_j)^2 \geq -(y - p_M)^2 - (y - p_j)^2 \geq -(\bar{y} - p_M)^2 - (\bar{y} - p_j)^2 \geq -\sum_i(\bar{y} - p_i)^2 \geq -G$, so $-(y - p_j)^2 \geq -G$, as desired.

Perfect. Then party M chooses $y = \frac{1}{2}(p_M + p_j)$, $g_j = (y - p_j)^2$, and $g_M = G - (y - p_j)^2$. Hence

$$u_M = -2(y - p_M)^2 + G = -\frac{1}{2}(p_j - p_M)^2 + G.$$

M will choose j to be the party that is closest ideologically.

Let $y_M^* := \frac{1}{2}(p_M + p_j)$ for future reference.

Note that M gets utility $u_M > 0$ but her partner gets 0 utility.

Stage 2: R is the formateur

Two cases. *First*, L is closest to M.

In that case L gets 0 utility if R's proposal fails, and $y_M^* = \frac{1}{2}(p_M + p_L)$.

If R proposes to L, by the same argument as before, R chooses $y = \frac{1}{2}(p_R + p_L)$ and $u_R = -\frac{1}{2}(p_R - p_L)^2 + G$.

If R proposes to M, her problem is to choose y, g_R, g_M to

$$\begin{aligned}
\max \quad & -(y - p_R)^2 + g_R \\
\text{s.t.} \quad & -(y - p_M)^2 + g_M \geq -\frac{1}{2}(p_M - p_L)^2 + G \\
& g_L + g_R \leq G \\
& 0 \leq g_L, g_R.
\end{aligned}$$

The constraints must bind, so $g_R = G - g_L$, $g_L = -\frac{1}{2}(p_M - p_L)^2 + G + (y - p_M)^2$.

The problem becomes

$$\begin{aligned} \max \quad & -(y-p_R)^2 + G + \frac{1}{2}(p_M - p_L)^2 - G - (y-p_M)^2 \\ \text{s.t.} \quad & -\frac{1}{2}(p_M - p_L)^2 + G + (y - p_M)^2 \leq G. \end{aligned}$$

Ignoring the constraint we get an upper bound on what R gets, which is $-\frac{1}{2}(p_M - p_R)^2 + \frac{1}{2}(p_M - p_L)^2$.

If she proposes to L, R gets $-\frac{1}{2}(p_R - p_L)^2 + G$. By taking G large enough we get that R prefers proposing to L.

So, in this case (L is closest to M), R proposes to L, chooses $y = \frac{1}{2}(p_L + p_R)$, and gives L zero utility.

Second case. R is closest to M. In this case L is even cheaper, so again R proposes to her. R chooses y, g_R, g_L to

$$\begin{aligned} \max \quad & -(y-p_R)^2 + g_R \\ \text{s.t.} \quad & -(y-p_L)^2 + g_L \geq -\left(\frac{p_M+p_R}{2} - p_L\right)^2 \\ & g_R + g_L \leq G \\ & 0 \leq g_R, g_L. \end{aligned}$$

We have $g_R = G - g_L$, so we can re-write this as follows

$$\begin{aligned} \max \quad & -(y-p_R)^2 + G - g_L \\ \text{s.t.} \quad & g_L \geq (y-p_L)^2 - \left(\frac{p_M+p_R}{2} - p_L\right)^2 \qquad (1) \\ & g_L \geq 0 \qquad (2) \\ & g_L \leq G. \end{aligned}$$

Either (1) or (2) must bind. We get that $y = \frac{1}{2}(p_M + p_R)$ and $g_L = 0$ — it's cheaper to pay L in policy than in cabinet positions.

Stage 1: L is the formateur

In the first stage, R is in a very good bargaining position, and M is in a terrible one, since R will exclude her if L fails. So, M is cheap, and is the closest party. Therefore L chooses M. [This is of course not a formal argument.]

There are many cases. In every case we get that L chooses a policy in $[p_L, p_M)$. In some cases $g_M > 0$, but in every case M receives negative utility.

Conclusion. As expected, the largest and smallest parties form a coalition. The policy is somewhere between their ideal points.

What happens with more than three parties?

Natural question. What happens with more than three parties? I think that the logic extends.

If G is large enough (equivalently, if parties do not care that much about policy) then two things happen: the formateur can buy off any party, and appropriates most of the rents from government.

This implies that the party in line to be the next formateur has a huge continuation value, so the first formateur will never try to include it. She will form a minimal coalition with the rest of the parties.

Which ones? Well, the ones closest ideologically are cheaper, but as we saw the continuation values can be complicated.

What happens if G is small?

The main intuition breaks down. The argument really depends on G being relatively large.

Example. Suppose $G = 0$, $w_L > w_M > w_R$, $p_L = -1$, $p_M = \frac{1}{2}$, $p_R = 1$, and the caretaker government chooses $y = -1$. (Or assume that if the three stages fail, L can form a minority government and choose her ideal policy.)

- In the third stage, R chooses $y = 1$ and gets M on board.
- In the second stage, M chooses $y = \frac{1}{2}$ and gets L on board.
- In the first stage, L chooses $y = \frac{1}{2}$ and gets M on board.

Takeaways from this example:

- The first and second largest parties form a coalition.
- The "junior partner" dictates policy.
- All this despite the huge ex ante advantage of L.

Dziuda and Loeper (2016)

- Two players, L, R.
- Two dates, $t = 1, 2$.
- A state $\theta_t \sim U[-a, a]$ revealed at the start of each date.
- A policy choice $x_t \in \{-1, 1\}$ chosen by unanimity. Otherwise, the status quo x_{t-1} is implemented.
- Stage payoffs: $u_L(x, \theta) = -(x - (\theta - p))^2$ and $u_R(x, \theta) = -(x - (\theta + p))^2$.
- In words, both want to match the state, but are biased. L's ideal policy is $\theta_t - p$ and R's ideal policy is $\theta_t + p$. The parameter $p > 0$ measures polarization, i.e., the extent of disagreement.
- There are only two actions, so if L only cares about the present, she prefers $x_t = 1$ iff $-(1 - (\theta_t - p))^2 \geq -(-1 - (\theta_t - p))^2$, i.e., iff $\theta_t \geq p$.
- If R only cares about the present, she prefers $x_t = 1$ iff $\theta_t \geq -p$.
- In sum, if $\theta_t \in [-p, p]$ they disagree, but otherwise they agree.

Enter dynamic considerations

They know that at time $t = 2$ the policy x_1 will be maintained against the will of one of them iff $\theta_2 \in [-p, p]$.

Suppose that $x_0 = -1$. This is the policy that L likes most of the time, so she starts with an advantage.

Suppose that $\theta_1 > p$.

At time $t = 1$, L prefers $x_1 = 1$ to the status quo. But if she agrees to implement $x_1 = 1$, she loses her advantage in the next period.

Concretely, her expected utility at time 1 is

$$\begin{aligned}
\mathbb{E}u_L &= -(x_1 - (\theta_1 - p))^2 - \mathbb{E}[(x_2 - (\theta_2 - p))^2] \\
&= -(x_1 - (\theta_1 - p))^2 - \int_{-a}^{a} (x_2 - (\theta - p))^2 \frac{1}{2a}\, d\theta \\
&= -(x_1 - (\theta_1 - p))^2 \\
&\quad - \int_{-a}^{p} (-1 - (\theta - p))^2 \frac{1}{2a}\, d\theta - \int_{-p}^{p} (x_1 - (\theta - p))^2 \frac{1}{2a}\, d\theta - \int_{-a}^{a} (1 - (\theta - p))^2 \frac{1}{2a}\, d\theta.
\end{aligned}$$

So, she prefers $x_1 = 1$ to $x_1 = -1$ iff

$$-(1-(\theta_1-p))^2 - \int_{-p}^{p}(1-(\theta-p))^2\frac{1}{2a}\,d\theta \geq -(-1-(\theta_1-p))^2 - \int_{-p}^{p}(-1-(\theta-p))^2\frac{1}{2a}\,d\theta$$

$$4(\theta_1-p) - \int_{-p}^{p} 4(\theta-p)\frac{1}{2a}\,d\theta \geq 0$$

$$\boxed{\theta_1 \geq p + \frac{p^2}{2a}.}$$

As expected, L needs a stronger reason at time 1 to accept $x_1 = 1$, because for $\theta_1 \in (p, p+\frac{p^2}{2a})$, even though she would be better off accepting the reform, she doesn't want to lose her advantage tomorrow.

Note that this is Pareto suboptimal. Both would be better off choosing $x_1 = 1$ for that θ_t.