Section 6

Electoral Competition with Valence Advantage

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Plan

We will derive the results from the model that we saw in lecture. These are:

- Adding a bit of valence advantage pushes the advantaged party to the center, and pushes the disadvantaged party to the extreme.
- If valence is sufficiently high, this force reverses for the advantaged party — it starts moving towards its ideal point as its advantage increases.

It's enough if we just do the first one. This is a complicated model to analyze!

If we have time, we can see how adding a common taste shock to voters' preferences lets us get a simple expression for the probability of winning of a party.

Groseclose (2001)

We will maintain the notation we were using in lecture, but we follow the paper.

Reminder.

- We have two candidates, A and B, with ideal points -1 and 1 respectively.
  - They choose policy platforms x_A and x_B.
- Policy preferences are given by $u(x,t) = -l(|x-t|)$, where
  - x is the policy implemented,
  - t is the individual's ideal point, and
  - l is a loss function such that $l'(0) = 0$, $l' > 0$, $l'' \geq 0$.
    - This implies that $u(\cdot, t)$ is concave and single-peaked at t.
    - Canonical case: $l(x) = x^2$, which implies $u(x,t) = -(x-t)^2$.

- The median voter's ideal point, t^M, is unknown at the time of choosing policy platforms.
  - We have $t^M \sim G$, i.e., $\Pr(t^M \leq x) = G(x)$, where
    - G is a CDF whose support includes $[-1, 1]$,
    - is symmetric around 0, and
    - $g = G'$ is its density.
- A voter of type t votes for A if $u(x_A, t) + v > u(x_B, t)$, where
  - $v \geq 0$ is A's valence advantage.
- Candidate's payoffs:
  - If candidate $i \in \{A, B\}$ wins, her payoff is $b + u(x_i, t_i)$, where
    - $b > 0$ is the rent from holding office, and
    - t_i is her ideal point.
  - If she loses, she gets $u(x_{-i}, t_i)$, where
    - x_{-i} is the policy platform of the other candidate.

Who wins?

A wins if $u(x_A, t) + v > u(x_B, t)$ for more than half of the voters.

- If $u(x_A, t^M) + v > u(x_B, t^M)$, it also happens for any $t \leq t^M + \epsilon$ assuming $x_A \leq x_B$, so A wins.
- Similarly, if $u(x_A, t^M) + v < u(x_B, t^M)$, then it happens for any $t \geq t^M - \epsilon$, so B wins.
- If $u(x_A, t^M) + v = u(x_B, t^M)$ then there is a tie, so each candidate wins with probability $\frac{1}{2}$.
  - Note that if $x_A \neq x_B$ this happens with probability zero, so we can ignore it, and if $v > 0$ this can't happen for $x_A = x_B$.
  - The only case that we may need to consider is $x_A = x_B$ and $v = 0$.
    - We know from last week that if $v = 0$ and $bg(0) < l'(1)$ then in equilibrium we have $x_A < x_B$, so let's assume $bg(0) < l'(1)$ and not worry about this.

In sum, A wins if and only if $u(x_A, t^M) + v > u(x_B, t^M)$, i.e., the median voter decides.

A wins if and only if $u(x_A, t^M) + v > u(x_B, t^M)$.

Let's assume that $x_A \leq x_B$. You can check that this must be the case in equilibrium, but let's just assume it.

Let t^* be such that $u(x_A, t^*) + v = u(x_B, t^*)$.
Then A wins iff $t^M < t^*$, so $\Pr(A \text{ wins}) = G(t^*)$.

Example. Let $l(x) = x^2$, so $u(x, t^*) = -(x - t^*)^2$.

A wins if and only if $u(x_A, t^M) + v > u(x_B, t^M)$.

Let's assume that $x_A \leq x_B$. You can check that this must be the case in equilibrium, but let's just assume it.

Let t^* be such that $u(x_A, t^*) + v = u(x_B, t^*)$.
Then A wins iff $t^M < t^*$, so $\Pr(A \text{ wins}) = G(t^*)$.

Example. Let $l(x) = x^2$, so $u(x, t^*) = -(x - t^*)^2$.

$$\begin{aligned}
u(x_A, t^*) + v &= u(x_B, t^*) \\
-(x_A - t^*)^2 + v &= -(x_B - t^*)^2 \\
v &= (x_A - t^*)^2 - (x_B - t^*)^2 \\
v &= (x_A - x_B)(x_A + x_B - 2t^*) \\
v &= \frac{1}{2}(x_B - x_A)\left(t^* - \frac{x_A + x_B}{2}\right) \\
\frac{v}{2(x_B - x_A)} &= t^* - \frac{x_A + x_B}{2} \\
t^* &= \frac{x_A + x_B}{2} + \frac{v}{2(x_B - x_A)}.
\end{aligned}$$

Can $x_A = x_B$ be an equilibrium if $v > 0$?

Note that B loses for sure, so her payoff is $u(x_A, 1)$.

- If she deviates, maybe with some probability the median voter will prefer her (despite A's valence advantage), so she can never lose.
- If v is small or t^M can be very large with positive probability, this will happen for any $x_B > x_A$.
- We can also just assume that if B is indifferent she will choose the platform closest to her ideal point.

So, if x_A, x_B is a pure-strategy equilibrium then $x_A < x_B$.

A's problem

A's expected utility is

$$\begin{aligned} U_A(x_A, x_B) &= \Pr(A \text{ wins})(b + u(x_A, -1)) + \Pr(A \text{ loses})u(x_B, -1) = \\ &= G(t^*)(b + u_A(x_A)) + (1 - G(t^*))u_A(x_B) = \\ &= G(t^*)(b + u_A(x_A) - u_A(x_B)) + u_A(x_B). \end{aligned}$$

Taking x_B as given, we have

$$\frac{\partial}{\partial x_A} U_A(x_A, x_B) = g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(x_A) - u_A(x_B)) + G(t^*)u'_A(x_A).$$

If $x_A < x_B$ is an equilibrium, the FOC $\frac{\partial}{\partial x_A} U_A(x_A, x_B) = 0$ must hold (because A can move to the left and to the right).

B's problem

B's expected utility is

$$\begin{aligned} U_B(x_A, x_B) &= \Pr(A \text{ wins}) u(x_A, 1) + (1 - \Pr(A \text{ loses}))(b + u(x_B, 1)) = \\ &= G(t^*) u_B(x_A) + (1 - G(t^*))(b + u_B(x_B)) = \\ &= (1 - G(t^*))(b + u_B(x_B) - u_B(x_A)) + u_B(x_A). \end{aligned}$$

Taking x_A as given, we have

$$\frac{\partial}{\partial x_B} U_B(x_A, x_B) = -g(t^*) \frac{\partial t^*}{\partial x_B} (b + u_B(x_B) - u_B(x_A)) + (1 - G(t^*)) u'_B(x_B).$$

The FOC $\frac{\partial}{\partial x_B} U_B(x_A, x_B) = 0$ must hold.

Equilibrium

If the strategies (x_A, x_B) form an equilibrium then we must have

$$g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(x_A) - u_A(x_B)) = -G(t^*)u'_A(x_A), \tag{1}$$

$$g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(x_B) - u_B(x_A)) = (1 - G(t^*))u'_B(x_B). \tag{2}$$

Now,

- Equation (1) defines implicitly A's best response $\tilde{x}_A(x_B, v)$.
- Equation (2) defines implicitly B's best response $\tilde{x}_B(x_A, v)$.
- In equilibrium, we must have

$$x_A^* = \tilde{x}_A(x_B^*, v) = \tilde{x}_A(\tilde{x}_B(x_A^*, v), v).$$

We will use this formula to derive comparative statics $\frac{\partial x_A^*}{\partial v}$, i.e., what happens with A's position as v increases.

Differentiating both sides of $x_A^* = \tilde{x}_A(\tilde{x}_B(x_A^*, v), v)$ we get

$$\frac{\partial x_A^*}{\partial v} = \frac{\partial \tilde{x}_A}{\partial x_B}\left(\frac{\partial \tilde{x}_B}{\partial x_A}\frac{\partial x_A^*}{\partial v} + \frac{\partial \tilde{x}_B}{\partial v}\right) + \frac{\partial \tilde{x}_A}{\partial v}.$$

Solving for $\frac{\partial x_A^*}{\partial v}$, we obtain

$$\left(1 - \frac{\partial \tilde{x}_A}{\partial x_B}\frac{\partial \tilde{x}_B}{\partial x_A}\right)\frac{\partial x_A^*}{\partial v} = \frac{\partial \tilde{x}_A}{\partial x_B}\frac{\partial \tilde{x}_B}{\partial v} + \frac{\partial \tilde{x}_A}{\partial v}.$$

We want to prove that $\frac{\partial}{\partial v}x_A^*(x_B^*, 0) > 0$, i.e., that A moderates (moves to the center) when she gets a small valence advantage.

We will have to calculate x_A^*, x_B^* for $v = 0$, and then the four partial derivatives $\frac{\partial \tilde{x}_A}{\partial x_B}$, $\frac{\partial \tilde{x}_B}{\partial x_A}$, $\frac{\partial \tilde{x}_A}{\partial v}$ and $\frac{\partial \tilde{x}_B}{\partial v}$ evaluated at $x_A = x_A^*$, $x_B = x_B^*$, $v = 0$.

The equilibrium for $v = 0$

First, we need to calculate the equilibrium for $v = 0$, which is the model we analyzed last week.

We will assume that $l(x) = x^2$, so $u(x,t) = -(x-t)^2$.

The equilibrium necessary condition is

$$g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(x_A) - u_A(x_B)) = -G(t^*)u_A'(x_A),$$
$$g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(x_B) - u_B(x_A)) = (1 - G(t^*))u_B'(x_B).$$

We will assume that the equilibrium is symmetric around 0. With G uniform it's easy to prove that this must be the case.

Let $z = x_A^*$ be the equilibrium platform for $v = 0$, so $X_B^* = -z$.

We have $t^* = 0$ and thus $G(t^*) = \frac{1}{2}$, so the first FOC becomes

$$
\begin{aligned}
g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(x_A) - u_A(x_B)) &= -G(t^*)u_A'(x_A) \\
g(0)\frac{1}{2}(b + u_A(z) - u_A(-z)) &= -\frac{1}{2}u_A'(z) \\
g(0)\frac{1}{2}(b - (z+1)^2 + (-z+1)^2) &= \frac{1}{2}2(z+1) \\
g(0)(b - 4z) &= 2(z+1) \\
(-4g(0) - 2)z &= 2 - g(0)b
\end{aligned}
$$

$$\boxed{z = -\frac{2 - g(0)b}{2(1 + 2g(0))}.}$$

Note that we were assuming $g(0)b < l'(1) = 2$, so $z < 0$.

The partial derivatives

Now we proceed to calculate the derivatives $\frac{\partial \tilde{x}_A}{\partial x_B}$, $\frac{\partial \tilde{x}_B}{\partial x_A}$, $\frac{\partial \tilde{x}_A}{\partial v}$ and $\frac{\partial \tilde{x}_B}{\partial v}$ evaluated at $x_A = z$, $x_B = -z$, $v = 0$.

Recall that $\tilde{x}_A(x_B, v)$, $\tilde{x}_B(x_A, v)$ and $t^*(x_A, x_B)$ were given by

$$
\begin{aligned}
g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(\tilde{x}_A) - u_A(x_B)) &= -G(t^*)u_A'(\tilde{x}_A),\\
g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(\tilde{x}_B) - u_B(x_A)) &= (1 - G(t^*))u_B'(\tilde{x}_B),\\
t^* &= \frac{x_A + x_B}{2} + \frac{v}{2(x_B - x_A)}.
\end{aligned}
$$

CALCULATING $\frac{\partial \tilde{x}_A}{\partial x_B}$

We differentiate both sides of the equation

$$g(t^*)\frac{\partial t^*}{\partial x_A}(b + u_A(\tilde{x}_A) - u_A(x_B)) = -G(t^*)u'_A(\tilde{x}_A)$$

with respect to x_B and get (ignoring terms that will disappear when setting $v = 0$)

$$g(t^*)\frac{\partial t^*}{\partial x_A}\left(u'_A(\tilde{x}_A)\frac{\partial \tilde{x}_A}{\partial x_B} - u'_A(x_B)\right) = -g(t^*)\frac{1}{2}\left(\frac{\partial \tilde{x}_A}{\partial x_B} + 1\right)u'_A(\tilde{x}_A) - G(t^*)u''_A(\tilde{x}_A)\frac{\partial \tilde{x}_A}{\partial x_B}.$$

We set $\tilde{x}_A = z$, $x_B = -z$, $v = 0$, thus $t^* = 0$, and get

$$g(0)\frac{1}{2}\left(u'_A(z)\frac{\partial \tilde{x}_A}{\partial x_B} - u'_A(-z)\right) = -g(0)\frac{1}{2}\left(\frac{\partial \tilde{x}_A}{\partial x_B} + 1\right)u'_A(z) + \frac{\partial \tilde{x}_A}{\partial x_B}.$$

Rearranging, we get

$$\left(g(0)\frac{1}{2}u'_A(z) + g(0)\frac{1}{2}u'_A(z) - 1\right)\frac{\partial \tilde{x}_A}{\partial x_B} = g(0)\frac{1}{2}u'_A(-z) - g(0)\frac{1}{2}u'_A(z),$$

so simplifying we arrive at

$$\boxed{\frac{\partial \tilde{x}_A}{\partial x_B} = -\frac{2g(0)z}{1 + 2g(0)(z+1)}.}$$

Calculating $\frac{\partial \tilde{x}_B}{\partial x_A}$

We differentiate both sides of the equation

$$g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(\tilde{x}_B) - u_B(x_A)) = (1 - G(t^*))u_B'(\tilde{x}_B)$$

with respect to x_A and get (ignoring terms that will disappear when setting $v = 0$)

$$g(t^*)\frac{\partial t^*}{\partial x_B}\left(u_B'(\tilde{x}_B)\frac{\partial \tilde{x}_B}{\partial x_A} - u_B'(x_A)\right) = -g(t^*)\frac{1}{2}\left(\frac{\partial \tilde{x}_B}{\partial x_A} + 1\right)u_B'(\tilde{x}_B) + (1 - G(t^*))u_B''(\tilde{x}_B)\frac{\partial \tilde{x}_B}{\partial x_A}.$$

We set $\tilde{x}_A = z$, $x_B = -z$, $v = 0$, thus $t^* = 0$, and get

$$g(0)\frac{1}{2}\left(u_B'(-z)\frac{\partial \tilde{x}_B}{\partial x_A} - u_B'(z)\right) = -g(0)\frac{1}{2}\left(\frac{\partial \tilde{x}_B}{\partial x_A} + 1\right)u_B'(-z) - \frac{\partial \tilde{x}_B}{\partial x_A}.$$

Rearranging, we get

$$\left(g(0)\frac{1}{2}u'_B(-z) + g(0)\frac{1}{2}u'_B(-z) + 1\right)\frac{\partial \tilde{x}_B}{\partial x_A} = g(0)\frac{1}{2}u'_B(z) - g(0)\frac{1}{2}u'_B(-z),$$

so simplifying we arrive at

$$\boxed{\frac{\partial \tilde{x}_B}{\partial x_A} = -\frac{2g(0)z}{1 + 2g(0)(z+1)}.}$$

The surprising result is that

$$\frac{\partial \tilde{x}_A}{\partial x_B}(-z, 0) = \frac{\partial \tilde{x}_B}{\partial x_A}(z, 0).$$

Calculating $\frac{\partial \tilde{x}_A}{\partial v}$

We differentiate both sides of the equation

$$g(t^*)\frac{\partial t^*}{\partial x_A}(b+u_A(\tilde{x}_A)-u_A(x_B)) = -G(t^*)u_A'(\tilde{x}_A) \qquad (*)$$

with respect to v and get (ignoring terms that will disappear when setting $v=0$)

$$g(t^*)\frac{1}{2(x_B-\tilde{x}_A)^2}(b+u_A(\tilde{x}_A)-u_A(x_B)) + g(t^*)\frac{\partial t^*}{\partial x_A}u_A'(\tilde{x}_A)\frac{\partial \tilde{x}_A}{\partial v} = \\ -g(t^*)\left(\frac{1}{2(x_B-\tilde{x}_A)} + \frac{1}{2}\frac{\partial \tilde{x}_A}{\partial v}\right)u_A'(\tilde{x}_A) - G(t^*)u_A''(\tilde{x}_A)\frac{\partial \tilde{x}_A}{\partial v}.$$

We set $\tilde{x}_A = z$, $x_B = -z$, $v = 0$, thus $t^* = 0$, and get

$$g(0)\frac{1}{8z^2}(b+u_A(z)-u_A(-z)) + g(0)\frac{1}{2}u_A'(z)\frac{\partial \tilde{x}_A}{\partial v} = -g(0)\left(\frac{1}{4z}+\frac{1}{2}\frac{\partial \tilde{x}_A}{\partial v}\right)u_A'(z) + \frac{1}{2}2\frac{\partial \tilde{x}_A}{\partial v}.$$

Replacing $g(0)(b + u_A(z) - u_A(-z)) = -u'_A(z)$ from $(*)$ and rearranging, we get

$$(g(0)u'_A(z) - 1)\frac{\partial \tilde{x}_A}{\partial v} = \frac{1}{4z}\left(\frac{1}{2z} - g(0)\right)u'_A(z),$$

so simplifying we arrive at

$$\boxed{\frac{\partial \tilde{x}_A}{\partial v} = \frac{(1+z)(1-2g(0)z)}{4z^2(1+2g(0)(z+1))}}.$$

Calculating $\frac{\partial \tilde{x}_B}{\partial v}$

We differentiate both sides of the equation

$$g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(\tilde{x}_B) - u_B(x_A)) = (1 - G(t^*))u'_B(\tilde{x}_B) \qquad (*)$$

with respect to v and get (ignoring terms that will disappear when setting $v = 0$)

$$\begin{aligned} &- g(t^*)\frac{1}{2(x_B - \tilde{x}_A)^2}(b + u_B(\tilde{x}_B) - u_B(x_A)) + g(t^*)\frac{\partial t^*}{\partial x_B}u'_B(\tilde{x}_B)\frac{\partial \tilde{x}_B}{\partial v} = \\ &\qquad - g(t^*)\left(\frac{1}{2(x_B - \tilde{x}_A)} + \frac{1}{2}\frac{\partial \tilde{x}_B}{\partial v}\right)u'_B(\tilde{x}_B) + (1 - G(t^*))u''_B(\tilde{x}_B)\frac{\partial \tilde{x}_B}{\partial v}. \end{aligned}$$

We set $\tilde{x}_A = z$, $x_B = -z$, $v = 0$, thus $t^* = 0$, and get

$$-g(0)\frac{1}{8z^2}(b+u_B(-z)-u_B(z))+g(0)\frac{1}{2}u'_B(z)\frac{\partial \tilde{x}_B}{\partial v} = -g(0)\left(\frac{1}{4z} + \frac{1}{2}\frac{\partial \tilde{x}_B}{\partial v}\right)u'_B(-z)+\frac{1}{2}(-2)\frac{\partial \tilde{x}_B}{\partial v}.$$

Replacing $g(0)(b + u_A(z) - u_A(-z)) = u'_B(-z)$ from $(*)$ and rearranging, we get

$$(g(0)u'_B(-z) + 1)\,\frac{\partial \tilde{x}_B}{\partial v} = \frac{1}{8z^2}(1 - 2g(0)z)u'_B(-z),$$

so simplifying we arrive at

$$\boxed{\frac{\partial \tilde{x}_B}{\partial v} = \frac{(1+z)(1-2g(0)z)}{4z^2(1+2g(0)(z+1))}}.$$

Again, surprisingly we get

$$\frac{\partial \tilde{x}_A}{\partial v}(-z,0) = \frac{\partial \tilde{x}_B}{\partial v}(z,0).$$

Finally, $\frac{\partial x_A^*}{\partial v}$

We had this expression:

$$\left(1-\frac{\partial \tilde{x}_A}{\partial x_B}\frac{\partial \tilde{x}_B}{\partial x_A}\right)\frac{\partial x_A^*}{\partial v}=\frac{\partial \tilde{x}_A}{\partial x_B}\frac{\partial \tilde{x}_B}{\partial v}+\frac{\partial \tilde{x}_A}{\partial v}.$$

Using $\frac{\partial \tilde{x}_A}{\partial x_B}=\frac{\partial \tilde{x}_B}{\partial x_A}$ and $\frac{\partial \tilde{x}_A}{\partial v}=\frac{\partial \tilde{x}_B}{\partial v}$, this simplifies to

$$\left(1-\left(\frac{\partial \tilde{x}_A}{\partial x_B}\right)^2\right)\frac{\partial x_A^*}{\partial v}=\left(1+\frac{\partial \tilde{x}_A}{\partial x_B}\right)\frac{\partial \tilde{x}_A}{\partial v}$$

$$\left(1-\frac{\partial \tilde{x}_A}{\partial x_B}\right)\frac{\partial x_A^*}{\partial v}=\frac{\partial \tilde{x}_A}{\partial v}.$$

Using $z=-\frac{2-g(0)b}{2(1+2g(0))}$ we can verify that $\frac{\partial \tilde{x}_A}{\partial x_B}<1$, and clearly $\frac{\partial \tilde{x}_A}{\partial v}>0$, so we get

$$\frac{\partial x_A^*}{\partial v}>0,$$

as we wanted to prove.

WHAT ABOUT $\frac{\partial x_B^*}{\partial v}$?

We can use the same strategy: $x_B^* = \tilde{x}_B(x_A^*, v) = \tilde{x}_B(\tilde{x}_A(x_B^*, v), v)$, so

$$\frac{\partial x_B^*}{\partial v} = \frac{\partial \tilde{x}_B}{\partial x_A}\left(\frac{\partial \tilde{x}_A}{\partial x_B}\frac{\partial x_B^*}{\partial v} + \frac{\partial \tilde{x}_A}{\partial v}\right) + \frac{\partial \tilde{x}_B}{\partial v},$$

and

$$\left(1 - \frac{\partial \tilde{x}_B}{\partial x_A}\frac{\partial \tilde{x}_A}{\partial x_B}\right)\frac{\partial x_B^*}{\partial v} = \frac{\partial \tilde{x}_B}{\partial x_A}\frac{\partial \tilde{x}_A}{\partial v} + \frac{\partial \tilde{x}_B}{\partial v}.$$

We already calculated these partial derivatives. Same reasoning:

$$\left(1 - \left(\frac{\partial \tilde{x}_A}{\partial x_B}\right)^2\right)\frac{\partial x_B^*}{\partial v} = \left(1 + \frac{\partial \tilde{x}_A}{\partial x_B}\right)\frac{\partial \tilde{x}_A}{\partial v},$$

$$\left(1 - \frac{\partial \tilde{x}_A}{\partial x_B}\right)\frac{\partial x_B^*}{\partial v} = \frac{\partial \tilde{x}_A}{\partial v},$$

and we get

$$\frac{\partial x_B^*}{\partial v} = \frac{\partial x_A^*}{\partial v} > 0.$$

Conclusion

When we give A *a bit* of valence advantage, i.e., we move from $v = 0$ to $v = \epsilon > 0$, both candidates move to the right.

- In the case of A, this means that she moderates, i.e., moves towards the center.
- In the case of B, she moves towards her ideal point, i.e., she becomes more extreme.

Next question. What happens when we give A *a lot* of valence advantage?

Answer. A moves to her ideal point, and B moves to the right of her ideal point (!).

When v is large

Recall that

$$t^* = \frac{x_A + x_B}{2} + \frac{v}{2(x_B - x_A)}.$$

We have

$$\frac{\partial t^*}{\partial x_A} = \frac{1}{2} + \frac{v}{2(x_B - x_A)^2},$$
$$\frac{\partial t^*}{\partial x_B} = \frac{1}{2} - \frac{v}{2(x_B - x_A)^2}.$$

It's always the case that $\frac{\partial t^*}{\partial x_A} > 0$. Hence $x_A \geq -1$ in equilibrium, because $x_A = -1$ is better than any $x_A < -1$.

B becomes an extremist

Let's assume that $v > 4$.

Suppose that $x_B \leq 1$ in equilibrium. Then $0 \leq x_B - x_A \leq 2$, hence $(x_B - x_A)^2 \leq 4$ and

$$\frac{\partial t^*}{\partial x_B} = \frac{1}{2} - \frac{v}{2(x_B - x_A)^2} < 0.$$

In that case, looking at

$$\frac{\partial U_B}{\partial x_B} = -g(t^*)\frac{\partial t^*}{\partial x_B}(b + u_B(x_B) - u_B(x_A)) + (1 - G(t^*))u_B'(x_B)$$

we see that $\frac{\partial U_B}{\partial x_B} > 0$ for any $x_B \leq 1$. Hence $x_B \leq 1$ cannot happen in equilibrium.

Conclusion. $x_B^* > 1$. In words, if $v > 4$, i.e., A's valence advantage is large, B adopts a policy more extreme than her ideal point!

A CHOOSES HER PREFERRED POLICY

Assume that $t^M \sim U\left[-\bar{v}, \bar{v}\right]$.

Assume that $v > (\bar{v}+1)^2$.

We have $-(\bar{v}+1)^2 + v > 0 \geq -(\bar{v}-x_B)^2$ for any x_B.

Hence if $t^M = \bar{v}$, then by choosing $x_A = -1$ A wins the election. Moreover, for any $t^M \leq \bar{v}$, this is also the case. Now, we are assuming $t^M \sim U\left[-\bar{v}, \bar{v}\right]$, hence A always wins the election with $x_A = -1$. This is also her ideal policy, hence she will choose it in equilibrium.

Conclusion. If v is large, assuming t^M uniform, in equilibrium A chooses her preferred policy.

We are done!