Section 5 – Electoral Competition

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Plan for today

Review electoral competition with policy motivations and uncertainty about the voters' positions in more detail.

Central insight. Downsian competition with

- Office-motivated candidates + no uncertainty $\rightarrow$ convergence.
- Office-motivated candidates + uncertainty $\rightarrow$ convergence.
- Policy-motivated candidates + no uncertainty $\rightarrow$ convergence.
- Policy-motivated candidates + uncertainty $\rightarrow$ divergence.

The essence of the argument

Let $P_A(x_A, x_B)$ the the probability that A wins if the policy commitments are x_A, x_B.

Candidate A chooses x_A to maximize

$$U(x_A) = P_A(x_A, x_B)(u_A(x_A) + b) + (1 - P_A(x_A, x_B))u_A(x_B).$$

Can $x_A = x_B$ be an equilibrium?

$$U'(x_B) = \frac{\partial}{\partial x_A} P_A(x_B, x_B)(u_A(x_B) - u_A(x_B) + b) + P_A(x_A, x_B)u'_A(x_B).$$

If b is very small (i.e., the candidate is more policy- than office-motivated), and $\frac{\partial}{\partial x_A} P_A(x_B, x_B)$ is small (i.e., there is significant uncertainty about voters' positions), then the first term is small, the second dominates, and $U'(x_B) < 0$, which implies that $x_A = x_B$ is not optimal for A, and thus $x_A = x_B$ can't be an equilibrium.

The details

We assume that $x_A \leq x_B$, because we saw in class that this is the case in any equilibrium.

Hence $P_A(x_A, x_B) = \Pr\left(t^M < \frac{x_A+x_B}{2}\right) = G\left(\frac{x_A+x_B}{2}\right)$.

In class we looked at A's problem. Let's look at B's problem now.

B has to maximize

$$U(x_B) := G\left(\frac{x_A + x_B}{2}\right) u_B(x_A) + \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right)(u_B(x_B) + b)$$

subject to $x_B \geq x_A$.

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subject to $x_B \geq x_A$.

We have

$$U'(x_B) = -\frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)(u_B(x_B) - u_B(x_A) + b) + \\ + \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u'_B(x_B).$$

FOC

If x_B is an interior optimum ($x_A < x_B < 1$) then it satisfies the FOC

$$U'(x_B) = -\frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)\left(u_B(x_B) - u_B(x_A) + b\right) + \\ + \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u_B'(x_B) = 0.$$

What about the SOC?

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$$U'(x_B) = -\frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)(u_B(x_B) - u_B(x_A) + b) +$$
$$+ \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u'_B(x_B) = 0.$$

What about the SOC?

$$U''(x_B) = -\frac{1}{4} g'\left(\frac{x_A + x_B}{2}\right)(u_B(x_B) - u_B(x_A) + b) -$$
$$- g\left(\frac{x_A + x_B}{2}\right) u'_B(x_B) + \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u''_B(x_B) <$$
$$< -\frac{1}{4} g'\left(\frac{x_A + x_B}{2}\right)(u_B(x_B) - u_B(x_A) + b) - g\left(\frac{x_A + x_B}{2}\right) u'_B(x_B) =$$
$$= -\frac{1}{4} g'\left(\frac{x_A + x_B}{2}\right) \frac{\left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u'_B(x_B)}{\frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)} - g\left(\frac{x_A + x_B}{2}\right) u'_B(x_B)$$

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$$U''(x_B) = -\frac{1}{4} g'\left(\frac{x_A + x_B}{2}\right) \frac{\left(1 - G\left(\frac{x_A+x_B}{2}\right)\right) u_B'(x_B)}{\frac{1}{2} g\left(\frac{x_A+x_B}{2}\right)} - \\ - g\left(\frac{x_A + x_B}{2}\right) u_B'(x_B) \propto \\ \propto -\frac{1}{2} g'\left(\frac{x_A + x_B}{2}\right) - \frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1 - G\left(\frac{x_A+x_B}{2}\right)}.$$

We want this to be negative.

We are happy to assume that $\frac{g(x)}{1-G(x)}$ is increasing. This means that $g'(x)(1 - G(x)) + g(x)^2 > 0$, i.e.,

$$-g'(x) < \frac{g(x)^2}{1 - G(x)}.$$

We can plug this inequality in

$$U''(x_B) \propto -\frac{1}{2}g'\left(\frac{x_A + x_B}{2}\right) - \frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1 - G\left(\frac{x_A+x_B}{2}\right)}.$$

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$$\begin{aligned} U''(x_B) \propto -\frac{1}{2}g'\left(\frac{x_A+x_B}{2}\right) - \frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1-G\left(\frac{x_A+x_B}{2}\right)} < \\ < \frac{1}{2}\frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1-G\left(\frac{x_A+x_B}{2}\right)} - \frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1-G\left(\frac{x_A+x_B}{2}\right)} = \\ = -\frac{1}{2}\frac{g\left(\frac{x_A+x_B}{2}\right)^2}{1-G\left(\frac{x_A+x_B}{2}\right)} < 0. \end{aligned}$$

So, if $U'(x_B) = 0$ then $U''(x_B) < 0$.

Implication: there is at most one solution to the FOC. Why? If there were two, they would be local maxima, but then there is a local minimum between them, contradiction.

Three possibilities:

- the optimum is interior, and hence $U'(x_B) = 0$,
- $x_B = 1$,
- $x_B = x_A$.

Can $x_B = 1$ be optimal?

Implication: there is at most one solution to the FOC. Why? If there were two, they would be local maxima, but then there is a local minimum between them, contradiction.

Three possibilities:

- the optimum is interior, and hence $U'(x_B) = 0$,
- $x_B = 1$,
- $x_B = x_A$.

Can $x_B = 1$ be optimal?

$$U'(1) = -\frac{1}{2}g\left(\frac{x_A+1}{2}\right)(u_B(1) - u_B(x_A) + b) + \\ + \left(1 - G\left(\frac{x_A+1}{2}\right)\right)u'_B(1) = \\ = -\frac{1}{2}g\left(\frac{x_A+1}{2}\right)(u_B(1) - u_B(x_A) + b) < 0.$$

B can't be happy setting $x_B = 1$. By moving a bit to the left he loses very little in policy but his probability of winning increases, so he is more likely to win office and implement his policy.

What about $x_B = x_A$?

What about $x_B = x_A$?

$$U'(x_A) = -\frac{1}{2}g(x_A)(u_B(x_A) - u_B(x_A) + b) + (1 - G(x_A))\,u'_B(x_A) =$$
$$= -\frac{1}{2}g(x_A)\,b + (1 - G(x_A))\,u'_B(x_A).$$

- If $U'(x_A) < 0$, then there can't be an interior local maximum, because if that is the case then there is also an interior local minimum, which we know is not the case. So if $U'(x_A) < 0$ then $x_B = x_A$ is the unique optimum.
- If $U'(x_A) \geq 0$, then there must be a local minimum, because there is a solution to the FOC, and we know that it must be unique. Moreover, if $U'(x_A) > 0$, then $x_B = x_A$ can't be optimal. If $U'(x_A) = 0$ then $x_B = x_A$ is the optimum.

So...

- If $U'(x_A) \leq 0$ then $x_B = x_A$ is the optimum.
- If $U'(x_A) > 0$ then the optimum is interior ($x_B > x_A$).

We know that the same applies to x_A symmetrically.

So, there are two cases for the equilibrium x_A, x_B:

- We have $x_A = x_B = x^*$, $U_A'(x^*) \geq 0$, $U_B'(x^*) \leq 0$.
- We have $x_A < x_B$, $U_A'(x_A) = 0$, $U_B'(x_B) = 0$.

First case. $(x_A = x_B = x^*)$ We have

$$U_A'(x^*) = \frac{1}{2} g\left(x^*\right)\left(u_A(x^*) - u_A(x^*) + b\right) + G\left(x^*\right) u_A'(x^*) \geq 0$$
$$U_B'(x^*) = -\frac{1}{2} g\left(x^*\right)\left(u_B(x^*) - u_B(x^*) + b\right) + \left(1 - G\left(x^*\right)\right) u_B'(x^*) \leq 0$$

So $\frac{1}{2} g\left(x^*\right) b \geq -G\left(x^*\right) u_A'(x^*)$ and $\frac{1}{2} g\left(x^*\right) b \geq \left(1 - G\left(x^*\right)\right) u_B'(x^*)$.

If $x^* = 0$ we get the condition $g(0)b \geq u_A'(0)$, which you saw in class $(l'(1) > bG'(0))$.

Second case. ($x_A < x_B$) We have

$$U_A'(x_A) = \frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)(u_A(x_A) - u_A(x_B) + b) + \\ + G\left(\frac{x_A + x_B}{2}\right) u_A'(x_A) = 0,$$

$$U_B'(x_B) = -\frac{1}{2} g\left(\frac{x_A + x_B}{2}\right)(u_B(x_B) - u_B(x_A) + b) + \\ + \left(1 - G\left(\frac{x_A + x_B}{2}\right)\right) u_B'(x_B) = 0.$$

If we assume symmetry, $x_A = x$, $x_B = -z$, this is

$$U_A'(z) = \frac{1}{2} g(0)(u_A(z) - u_A(-z) + b) + G(0) u_A'(z) = 0$$

$$U_B'(-z) = -\frac{1}{2} g(0)(u_B(-z) - u_B(z) + b) + (1 - G(0)) u_B'(-z) = 0$$

These are equivalent, so z solves

$$g(0)(-l(z+1) + l(-z+1) + b) = -l'(z+1).$$

Take $l(x) = x^2$. Then the equation for the symmetric equilibrium is

$$g(0)(-(z+1)^2 + (-z+1)^2 + b) = -2(z+1).$$

We can solve for z:

$$z^* = \frac{1}{4}\frac{g(0)b - 2}{g(0) - \frac{1}{2}}.$$

And we get the natural comparative statics: $\frac{\partial z^*}{\partial b} > 0$ and $\frac{\partial z^*}{\partial g(0)} > 0$.