Section 3 – Cheap talk

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Cheap talk

Timing:

1. Nature draws $\omega \sim \text{Uniform}[0,1]$.
2. Sender observes ω and chooses a report $r \in [0,1]$.
3. Receiver observes r and chooses $y \in \mathbb{R}$.

Payoffs:

$$u_S(y,\omega) = -(y-\omega-b)^2,$$
$$u_R(y,\omega) = -(y-\omega)^2.$$

We assume that $b > 0$, so the Sender always wants y to be greater than the Receiver.

In equilibrium the Receiver chooses $y = \mathbb{E}_R[\omega \mid r]$.

Strategies

A pure strategy for the sender is a function $r(\omega)$ that maps states to reports.

We need to specify beliefs for the receiver when she observes a message: $\mu(r) \in \Delta([0, 1])$.

By sequential rationality we know that in equilibrium $y = \mathbb{E}_R[\omega \mid r] = \int \omega \, d\mu(r)(\omega)$. This means that y is the expectation of ω assuming that its distribution is given by $\mu(r)$.

PBE requires that $\mu(r)$ is given by Bayes rule when r occurs in equilibrium, and that $r(\omega)$ is optimal given μ.

Equilibrium with three intervals

We saw in lecture how to construct equilibria with two intervals. Let's try to construct equilibria with three intervals.

We look for a strategy

$$r(\omega) = \begin{cases} r_1, & \text{if } \omega \in [0, \omega_1), \\ r_2, & \text{if } \omega \in [\omega_1, \omega_2), \\ r_3, & \text{if } \omega \in [\omega_2, 1]. \end{cases}$$

Given this strategy, the Receiver's beliefs must satisfy

$$\mu(r) = \begin{cases} \text{Uniform}[0, \omega_1), & \text{if } r = r_1, \\ \text{Uniform}[\omega_1, \omega_2), & \text{if } r = r_2, \\ \text{Uniform}[\omega_2, 1], & \text{if } r = r_3 \end{cases}$$

by Bayes rule.

For any other report r, the Receiver can believe whatever she wants.

If we understand the reports as recommendations, it makes sense that the Receiver's inference is as follows:

$$\mu(r) = \begin{cases} \text{Uniform}[0, \omega_1), & \text{if } r \in [0, \omega_1), \\ \text{Uniform}[\omega_1, \omega_2), & \text{if } r \in [\omega_1, \omega_2), \\ \text{Uniform}[\omega_2, 1], & \text{if } r \in [\omega_2, 1]. \end{cases}$$

The idea is that when the Receiver sees r, she cannot believe its exact value, but in equilibrium she is ready to believe that ω is in some interval, and r can communicate that interval in a very natural way: r points to the interval where it belongs.

We can think that the Receiver interprets r as one of three possible messages: "low", "medium" and "high" by looking at its size.

Given these beliefs, the Reciever's best response is:

$$y(r) = \begin{cases} \frac{\omega_1}{2}, & \text{if } r \in [0, \omega_1), \\ \frac{\omega_1+\omega_2}{2}, & \text{if } r \in [\omega_1, \omega_2), \\ \frac{\omega_2+1}{2}, & \text{if } r \in [\omega_2, 1]. \end{cases}$$

Therefore, we have to choose ω_1 and ω_2 (such that $0 < \omega_1 < \omega_2 < 1$) so that:

- when $\omega \in [0, \omega_1)$ the Sender prefers $y = \frac{\omega_1}{2}$ to $\frac{\omega_1+\omega_2}{2}$ and $\frac{\omega_2+1}{2}$,
- when $\omega \in [\omega_1, \omega_2)$ the Sender prefers $y = \frac{\omega_1+\omega_2}{2}$ to $\frac{\omega_1}{2}$ and $\frac{\omega_2+1}{2}$, and
- when $\omega \in [\omega_2, 1]$ the Sender prefers $y = \frac{\omega_2+1}{2}$ to $\frac{\omega_1}{2}$ and $\frac{\omega_1+\omega_2}{2}$.

Let's look at the borders of the intervals.

If $\omega = \omega_1 - \epsilon$, Sender prefers $\frac{\omega_1}{2}$ to $\frac{\omega_1+\omega_2}{2}$. Take $\epsilon \to 0$ and this implies that if $\omega = \omega_1$ then Sender (weakly) prefers $\frac{\omega_1}{2}$ to $\frac{\omega_1+\omega_2}{2}$.

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Hence, if $\omega = \omega_1$, Sender must be indifferent between $\frac{\omega_1}{2}$ and $\frac{\omega_1+\omega_2}{2}$.

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So, $\omega_2 = 2\omega_1 + 4b$.

By the same argument, if $\omega = \omega_2$ she must be indifferent between $\frac{\omega_1+\omega_2}{2}$ and $\frac{\omega_2+1}{2}$.

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So, $1 = 2\omega_2 - \omega_1 + 4b$.

Combining, we have $1 = 2(2\omega_1 + 4b) - \omega_1 + 4b = 3\omega_1 + 12b > 12b$. So $b < \frac{1}{12}$.

The Sender's bias has to be really small if an equilibrium with three intervals exists.

Suppose that $b < \frac{1}{12}$. We have to equations:

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$$\omega_2 = 2\omega_1 + 4b.$$

Suppose that $b < \frac{1}{12}$. We have to equations:

$$\begin{aligned} 1 &= 3\omega_1 + 12b, \\ \omega_2 &= 2\omega_1 + 4b. \end{aligned}$$

We get

$$\begin{aligned} \omega_1 &= \frac{1}{3} - 4b, \\ \omega_2 &= \frac{2}{3} - 4b. \end{aligned}$$

We got ω_1 and ω_2, so we determined our strategies and beliefs. Are they a PBE?

Recall that we have to check that:

- when $\omega \in [0, \omega_1)$ the Sender prefers $y = \frac{\omega_1}{2}$ to $\frac{\omega_1+\omega_2}{2}$ and $\frac{\omega_2+1}{2}$,
- when $\omega \in [\omega_1, \omega_2)$ the Sender prefers $y = \frac{\omega_1+\omega_2}{2}$ to $\frac{\omega_1}{2}$ and $\frac{\omega_2+1}{2}$, and
- when $\omega \in [\omega_2, 1]$ the Sender prefers $y = \frac{\omega_2+1}{2}$ to $\frac{\omega_1}{2}$ and $\frac{\omega_1+\omega_2}{2}$.

You can just check this by hand, but the key is that u_S satisfies a *single-crossing condition*: if $\omega < \omega'$, $y < y'$ and ω' prefers y to y' then ω does as well.

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The key to proving this is to note that if $\omega < \omega'$, $y < y'$ then

$$u_S(y', \omega') - u_S(y, \omega') > u_S(y', \omega) - u_S(y, \omega).$$

We can prove this by replacing $u_S(y, \omega) = -(y - \omega - b)^2$, in which case it simplifies to $(y' - y)(\omega' - \omega) > 0$, which is of course true, or we can note that the inequality means that

$$f(\omega) = u_S(y', \omega) - u_S(y, \omega)$$

is an increasing function. Now this means that

$$f'(\omega) = \frac{\partial}{\partial \omega} u_S(y', \omega) - \frac{\partial}{\partial \omega} u_S(y, \omega)$$

must be positive, so $\frac{\partial}{\partial \omega} u_S(y', \omega) > \frac{\partial}{\partial \omega} u_S(y, \omega)$. This means that $g(y) = \frac{\partial}{\partial \omega} u_S(y, \omega)$ must be increasing, which means that $g'(y) > 0$, i.e., $\frac{\partial^2}{\partial y \partial \omega} u_S(y, \omega) > 0$. In our case $\frac{\partial^2}{\partial y \partial \omega} u_S(y, \omega) = 2$.

Other things?

– Welfare
– Two senders
– Review bayesian persuasion

I recommend Chapter 4 of “Special Interest Politics” by Grosman and Helpman.