Section 12

Correction to Acemoglu-Robinson QJE (2000)

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Plan for today

- Say a bit more about MPEs
- Correct the analysis of Acemoglu-Robinson, following Acemoglu and Robinson (2017) "Why Did the WestExtend the Franchise? A Correction." Unpublished manuscript.

Markov Perfect Equilibrium

Suppose that we have a dynamic game of perfect information that is a repeated game, except that there is a changing *state* that affects payoffs.

So, suppose that we have n players, actions $x_i \in A_i$ and stage utilities $u_i(x_1, \ldots, x_n, s)$, where $s \in S$ is the state.

Actual utilities are $\sum_{t=0}^{\infty} \delta^t u_i(x_{1t}, \ldots, x_{nt}, s_t)$.

We assume that actions today can influence the next period state.

In a deterministic model, $s_{t+1} = f(x_{1t}, \ldots, x_{nt}, s_t)$.

In a stochastic model, $\Pr(s_{t+1} = s) = p(s \mid x_{1t}, \ldots, x_{nt}, s_t)$.

When choosing x_i, agent i thinks of the implications for her payoff $u_i(x_i, x_{-i}, s)$ today, and also considers the implications for the next period state, which affects her future payoffs.

Key assumption: everything payoff-relevant from the past is encapsulated in today's state.

Markov Perfect Equilibrium

A *Markov Perfect Equilibrium* (MPE) is a Subgame Perfect Equilibrium (SPE) where agents do not condition their strategies on the past, only on today's state. They don't have memory. (The word "Markov" is because the equilibrium dynamical system is a Markov process.)

They do condition their strategies on today's state, which is affected by past actions. So in this sense they do condition their actions on the past, but only on *payoff-relevant* consequences of past events.

For example, in the case of a repeated Prisoner's Dilemma, the state is constant, so the unique MPE is defect-defect always.

So, formally, a MPE is a profile of (mixed) strategies $\sigma_i : S \to \Delta(A_i)$ for each player that induce a SPE.

How to formalize extensive-form stage games? Allow $x_i \in A_i$ to be a function of x_j if i plays after j.

How to verify that strategies are a MPE?

Given a profile of Markov strategies $\sigma_i : S \to \Delta(A_i)$, how do we verify that they are an SPE?

We can use the **one-shot deviation principle**.

Take a state s and a player i. Assume that other players are choosing $x_j = \sigma_j(s)$ and in the future every player (including i) will play $\sigma_j(s)$. Verify that $x_i = \sigma_i(s)$ is a best response today.

Cool thing about MPEs: if in a SPE every player except i is using Markov strategies, i has a Markov best response. (Try to prove it yourself.)

Using this idea it's not hard to prove that if the sets of actions are finite then there is a MPE (possibly in mixed strategies). Idea: define a player (i, s) for each $i \in \{1, \ldots, n\}$ and $s \in S$ with utilities given by $u_{(i,s)}(x) = u_i(x_s, s) + \delta \sum_{s' \in S} p(s'|x_s, s) u_{(i,s')}(x, s')$, and apply Nash theorem.

Conceptual comment on Acemoglu-Robinson

The conceptual question is *why institutions?*, if we have an instrumental view of them.

If we think of democracy as rule by the median individual (abstracting from a thousand things that complicate this picture) and autocracy as rule by the powerful (in terms of assets, guns), we can ask:

- Why would the powerful yield power to the median individual?

A plausible answer is that the median individuals have strength in numbers.

But if they are powerful enough to choose institutions, why do they need to do it?

They could just choose their preferred policy. Why bother asking for democracy?

Acemoglu & Robinson's idea: democratization is a commitment device.

The poor *can* have strength in numbers, but only if they solve their collective action problem. When they do, they have power to dictate terms, but the elite cannot commit to let them choose policy in the future, unless they change institutions. The function of democracy (and institutions in general) is to constrain future collective decisions.

Acemoglu-Robinson QJE (2000)

Three regimes: Autocracy, Democracy, Revolution.

The poor can appropriate a share $\tilde{\mu} \in \{0, \mu\}$ of aggregate income in a revolution, with $\Pr(\tilde{\mu}_t = \mu) = q$ independently each date.

State space is $\{A, D, R\} \times \{0, \mu\}$.

Stage game:

1. The rich choose whether to democratize or not. (Only relevant in Autocracy.)
2. Both agents choose a tax rate $\tau_i \in [0, 1 - Z]$. (In Autocracy τ_r is implemented; in Democracy, τ_p is implemented.)
3. The poor chose whether to have a revolution or not. (Only relevant in Autocracy.)

The state transition is:

- Autocracy goes to Democracy if the rich democratize; if the poor revolt, goes to Revolution; otherwise, stays.
- Democracy and Revolution are absorbing.

Payoffs

In Autocracy and Democracy stage payoffs are

$$u_p = h_p + (1-\lambda)\tau(h_r - h_p),$$
$$u_r = h_r - \lambda\tau(h_r - h_p)$$

In Revolution they are $u_p = \mu\frac{H}{\lambda}$, $u_r = 0$, where $H = \lambda h_p + (1-\lambda)h_r$.

In Democracy the poor choose τ, so they choose $\tau = 1 - Z$. Hence

$$V^p(D) = h_p + (1-\lambda)(1-Z)(h_r - h_p),$$
$$V^r(D) = h_r - \lambda(1-Z)(h_r - h_p).$$

In Autocracy when $\tilde{\mu} = 0$ revolution is dominated (the worst that the poor can get if they don't revolt is $h_p > 0$), hence $\tau_r = 0$ and the rich do not democratize.

- If we relax the Markov assumption, we can get more flexibility. The poor can provide incentives for the rich to redistribute when they are weak ($\tilde{\mu} = 0$) if they credibly promise to revolt when they are strong ($\tilde{\mu} = \mu$).
  - Exercise: construct a non-Markov SPE where $\tau_r > 0$ always under Autocracy, and the poor revolt when $\tilde{\mu} = \mu$ iff the rich failed to redistribute in the past.

Autocracy with concessions

Let's construct a MPE where the rich never democratize, and the poor don't revolt.

In Autocracy when $\tilde{\mu} = \mu$ the poor need to prefer not to revolt.

To ease notation let $T_p := (1-\lambda)(h_r - h_p)$, so the stage payoff of the poor is $u_p = h_p + \tau T_p$.

Also, let $\bar{\tau} = 1 - Z$, so $\tau \in [0, \bar{\tau}]$.

If the poor don't revolt, they get $h_p + \tau T_p$ today and V tomorrow, where

$$\begin{aligned} V &= q\left[(1-\delta)(h_p + \tau T_p) + \delta V\right] + (1-q)\left[(1-\delta)h_p + \delta V\right] \\ &= (1-\delta)(h_p + q\tau T_p) + \delta V, \end{aligned}$$

so $V = h_p + q\tau T_p$. Hence, if they don't revolt they get

$$(1-\delta)(h_p + \tau T_p) + \delta(h_p + q\tau T_p) = h_p + (1-(1-q)\delta)\tau T_p.$$

If they revolt they get $\mu \frac{H}{\lambda}$.

Autocracy with concessions

Hence, we need

$$\mu \frac{H}{\lambda} \leq h_p + (1 - (1-q)\delta)\tau T_p.$$

If there is a $\tau \in [0, \bar{\tau}]$ that satisfies this condition, then the rich will choose the minimum τ that satisfies it, i.e., the τ^* that satisfies it with equality.

If that holds for some τ, then it also holds for $\bar{\tau}$, and conversely. Hence we have an equilibrium with autocracy iff

$$\boxed{\mu \frac{H}{\lambda} \leq h_p + (1 - (1-q)\delta)\bar{\tau} T_p.}$$

The rich do not need to democratize because they can appease the poor when they are strong by giving them a one-time transfer.

Democratization

Suppose that $\mu\frac{H}{\lambda} > h_p + (1-(1-q)\delta)\bar{\tau}T_p$, so concessions are not enough to prevent a revolution.

Will we have democratization for sure?

Consider strategies where the rich democratize, and the poor revolt if the rich do not democratize. Do they constitute a SPE?

(Note that the poor need to credibly threaten revolution to induce the rich to democratize, because democratizing is dominated for the rich if they think that the poor won't revolt.)

For any tax rate τ that the rich offer the poor, the poor have to prefer revolution.

If they don't revolt, they get $h_p + \tau T_p$ today and V tomorrow, where

$$\begin{aligned} V &= qV^p(D) + (1-q)\left[(1-\delta)h_p + \delta V\right] \\ &= q\left[h_p + \bar{\tau}T_p\right] + (1-q)(1-\delta)h_p + (1-q)\delta V, \end{aligned}$$

so

$$V = h_p + \frac{q\bar{\tau}}{1-(1-q)\delta}T_p.$$

Hence they get

$$h_p + \left((1-\delta)\tau + \frac{\delta q\bar{\tau}}{1-(1-q)\delta}\right)T_p.$$

If they revolt, they get $\mu\frac{H}{\lambda}$. The must prefer revolution for any τ, so the condition is

$$\mu\frac{H}{\lambda} \geq h_p + \left((1-\delta)\tau + \frac{\delta q\bar{\tau}}{1-(1-q)\delta}\right)T_p.$$

The RHS is increasing in τ, hence this is satisfied for any $\tau \in [0, \bar{\tau}]$ iff

$$\boxed{\mu\frac{H}{\lambda} \geq h_p + \left(1-\delta + \frac{\delta q}{1-(1-q)\delta}\right)\bar{\tau}T_p.}$$

What happens in between?

We have that if

$$\mu\frac{H}{\lambda} \leq h_p + (1-(1-q)\delta)(1-Z)T_p$$

then we have autocracy with concessions, and if

$$\mu\frac{H}{\lambda} \geq h_p + \left(1-\delta+\frac{\delta q}{1-(1-q)\delta}\right)\bar{\tau}T_p$$

then we have democratization.

There is a gap!

If

$$h_p + (1-(1-q)\delta)\bar{\tau}T_p < \mu\frac{H}{\lambda} < h_p + \left(1-\delta+\frac{\delta q}{1-(1-q)\delta}\right)\bar{\tau}T_p$$

then we can't have neither autocracy nor democratization for sure.

MPE in mixed strategies

Consider an equilibrium where the rich democratize with probability $\alpha \in (0,1)$, choose $\tau = \bar{\tau}$, and the poor revolt with probability $\beta \in (0,1)$.

Let $T_r := \lambda(h_r - h_p)$, so the rich get $h_r - \tau T_r$ if the tax rate is τ.

If the rich democratize, they get $h_r - \bar{\tau} T_r$. If they don't, they get, say, $f(\beta)$. We can obtain an expression for this, but it's enough to know that it's continuous (easy to verify).

For $\beta \in (0,1)$, we need $h_r - \bar{\tau} T_r = V(\beta)$. By the intermediate value theorem, it's enough to show that $f(0) > h_r - \bar{\tau} T_r > f(1)$.

Let's calculate $f(0)$ and $f(1)$.

- $f(0)$ is what the rich get in this equilibrium if the poor never revolt, i.e., hey get $h_r - \bar{\tau} T_r$ today, and tomorrow they get $h_r - q\bar{\tau} T_r$, so $f(0) = h_r - (1 - (1-q)\delta)\bar{\tau} T_r$.
- $f(1)$ is what they get if the poor revolt for sure, i.e., 0.

So, we need to check

$$h_r - (1 - (1-q)\delta)\bar{\tau} T_r > h_r - \bar{\tau} T_r > 0,$$

which is true.

The poor's decision

If the poor revolt, they get $\mu\frac{H}{\lambda}$.

If they don't, they get $h_p + \bar{\tau}T_p$ today and V tomorrow, where

$$V = q\left\{\alpha(h_p + \bar{\tau}T_p) + (1-\alpha)\left[\beta\mu\frac{H}{\lambda} + (1-\beta)\left[(1-\delta)(h_p + \bar{\tau}T_p) + \delta V\right]\right]\right\} + (1-q)\left[(1-\delta)h_p + \delta V\right].$$

We need $(1-\delta)(h_p + \bar{\tau}T_p) + \delta V = \mu\frac{H}{\lambda}$. Both equations determine V, so we can assume the second and check that the first is satisfied.

Assuming the second, the first becomes

$$(1-(1-q)\delta)V = q\left\{\alpha(h_p + \bar{\tau}T_p) + (1-\alpha)\mu\frac{H}{\lambda}\right\} + (1-q)(1-\delta)h_p.$$

This defines a continuous function $V(\alpha)$ increasing in α. We need $\alpha \in (0,1)$ such that $(1-\delta)(h_p + \bar{\tau}T_p) + \delta V(\alpha) = \mu\frac{H}{\lambda}$. It's enough to show that the $=$ is $<$ for $\alpha = 0$ and is $>$ for $\alpha = 1$.

The first inequality is (after doing some algebra)

$$h_p + (1 - \delta(1-q))\bar{\tau}T_p < \mu\frac{H}{\lambda}.$$

The second is

$$\mu\frac{H}{\lambda} < h_p + \left(1 - \delta + \frac{\delta q}{1-(1-q)\delta}\right)\bar{\tau}T_p.$$

These are exactly the conditions that defined the gap! Excellent.

Finally, we need to check that $\tau = \bar{\tau}$ is optimal for the rich.

If $\tau < \bar{\tau}$, then revolution is strictly greater than the continuation value for the poor, since we are assuming that with $\tau = \bar{\tau}$ they are indifferent. So, if the rich choose a smaller tax rate, the poor revolt for sure. But we know that there is no equilibrium in which the poor revolt for sure in this region of the parameter space.

Conclusion

We have to amend the Proposition in last lecture. There are two cutoffs, $\bar{q} < q^*$.

- If $q \leq \bar{q}$, there is democratization when $\tilde{\mu} = \mu$.
- If $\bar{q} < q < q^*$, there is democratization and revolutions with positive probability when $\tilde{\mu} = \mu$.
- If $q^* < q$, there is autocracy with a temporary concession when $\tilde{\mu} = \mu$.

Something nice about this: it predicts that communism should be rare but not impossible.

What about inequality?

The famous prediction of the Acemoglu-Robinson model is that democratization occurs for intermediate levels of inequality.

In the model that we studied we get democratization only when inequality is high. (If inequality is low, revolution is too costly relative to no redistribution, so the revolution threat is not credible.)

To get democratization only when inequality is low enough what AR do is to introduce repression.

They model repression as follows: if the rich choose repression, the poor cannot revolt, but everybody loses a fraction κ of income (this is the cost of repression).

If inequality is high, democracy is too costly for the rich, hence they prefer to repress when $\tilde{\mu} = \mu$. If repression is too costly relative to inequality, they democratize instead.

Exercise. Analyze the model with repression.